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# FRANKFORD ARSENAL

REPORT NO. R-1224



USE OF A HIGH-SPEED DIGITAL COMPUTER FOR  
ANALYSIS OF CATAPULT PERFORMANCE

BY

C. M. King

PROJECT TSI-15

PITMAN-DUNN LABORATORIES

FRANKFORD ARSENAL

PHILADELPHIA, PA.

October 1954

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REPORT R-1224

USE OF A HIGH-SPEED DIGITAL COMPUTER FOR ANALYSIS OF  
CATAPULT PERFORMANCE

PROJECT TS1-15

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## OBJECT

To investigate the use of high-speed computer techniques for the analysis of aircraft personnel catapult performance.

## SUMMARY

The high-speed computer has been used in the analysis and prediction of catapult performance. By means of an iterative numerical integration procedure, simultaneous records of thrust-time, velocity-time, and displacement-time can be generated for a given catapult system in a matter of a few minutes. Preliminary studies indicate that catapult performance with combinations of high frictional drag (such as rail friction) and high acceleration fields (caused by gravity and aircraft maneuvers) can be evaluated with these new techniques.

## AUTHORIZATION

00 471.4/2750, FA 471.4/5971, 24 Aug 45

## INTRODUCTION

This report derives a technique for using a high-speed digital computer to obtain the ballistic solution for an aircraft personnel catapult. Thrust, velocity, and stroke are given as functions of time. This technique may also be used for similar cartridge actuated devices. The procedure consists of using the equations of constraint of the system, i. e., the equations of motion, the energy equation, the equation of state or gas law, and the equation of burning, together with the initial conditions, in a stepwise, iterative, numerical integration procedure. In the solution thus generated, the following assumptions are made:

1. Gas leakage and initial friction are negligible.
2. A linear burning equation is used.
3. The equation of state for an ideal gas is used.
4. The heat loss is taken proportional to the kinetic energy of the ejected mass in a 1:3 ratio.
5. The ratio of specific heats,  $\gamma$ , is taken to be 1.25.
6. The multitube catapult can be treated as a constant bore catapult. The bore is determined by that tube in motion at the moment of maximum thrust.<sup>(1)</sup>
7. The retarding rail friction force,  $F_f$ , is proportional to the square of the stroke, i. e.,  $F_f = K(X - X_0)^2$ .<sup>(2)</sup>

## DEFINITION OF SYMBOLS

A = equivalent bore area	in. <sup>2</sup>
B = burning rate coefficient <sup>(3)</sup>	in./sec/psi
C = weight of charge	lb
$C_v$ = mean specific heat of gas at constant volume <sup>(4)</sup>	ft-lb/lb/°F
F = gas thrust (= PA)	lb
$F_f$ = rail friction retarding force	lb
$F^*$ = impetus of propellant (= $RT_g$ ) <sup>(3)</sup>	ft-lb/lb

<sup>(1)</sup>See Appendix I for conversion method.

<sup>(2)</sup>While this is a typical rail friction expression, the rail friction may be represented by any applicable algebraic function of X or V.

<sup>(3)</sup>These quantities do not appear directly, but their product,  $BF^*$ , appears in an adjusted constant, Q.

<sup>(4)</sup>These are nominal quantities that are used in the derivations of  $F^*$  and  $\gamma$ , but do not appear in the final equations.



$g$	= gravitational constant (= 32.17)	ft/sec <sup>2</sup>
$K$	= rail friction force coefficient where $F_f = K(X - X_0)^2$ <sup>(2)</sup>	lb/ft <sup>2</sup>
$L$	= equivalent stroke	ft
$M$	= ejected mass	slugs
$N$	= weight of propellant burned	lb
$P$	= gas pressure	psi
$Q$	= a ballistic parameter ( $= \frac{CBF^*}{WA}$ )	fps
$R$	= gas constant <sup>(4)</sup>	ft-lb/lb <sup>°</sup> F
$T$	= temperature of gas <sup>(4)</sup>	<sup>°</sup> F <sub>abs</sub>
$T_a$	= adiabatic flame temperature <sup>(4)</sup>	<sup>°</sup> F <sub>abs</sub>
$V$	= velocity of mass with respect to airplane, along path of ejection	fps
$\dot{V}$	= acceleration of mass with respect to airplane, along path of ejection	ft/sec <sup>2</sup>
$\dot{V}_{P/G}$	= acceleration of airplane with respect to earth	ft/sec <sup>2</sup>
$\dot{V}_{M/G}$	= acceleration of mass with respect to earth	ft/sec <sup>2</sup>
$v$	= gas volume (= 12AX)	in. <sup>3</sup>
$v_0$	= initial gas volume	in. <sup>3</sup>
$W$	= web thickness for a single perforated grain <sup>(5)</sup>	in.
$X$	= equivalent displacement (= travel + $X_0$ )	ft
$X_0$	= equivalent initial length ( $= \frac{v_0}{12A}$ )	ft
$X_m$	= equivalent muzzle length (= $X_0 + L$ )	ft

#### Greek Symbols

$\alpha$  = load factor just prior to ejection. Also, acceleration in g's of ejected mass with respect to airplane due to inertial and gravitational forces. <sup>(6)</sup>

<sup>(2)</sup> While this is a typical rail friction expression, the rail friction may be represented by any applicable algebraic function of  $X$  or  $V$ .

<sup>(4)</sup> These are nominal quantities that are used in the derivations of  $F^*$  and  $\gamma$ , but do not appear in the final equations.

<sup>(5)</sup> The equivalent web thickness for a 7-perforated grain is taken as 1.15 times the actual web thickness.

<sup>(6)</sup> By convention, a value of  $\alpha$  greater than zero represents a net force aiding the gas driving force,  $F$ .

$\beta$  = ratio of heat transferred to catapult to kinetic energy of ejected mass

$\gamma$  = ratio of specific heats ( $= C_p/C_v$ )

$\bar{\gamma}$  = a pseudo ratio of specific heats such that  $\bar{\gamma} - 1 = (1 + \beta)(\gamma - 1)$

$\theta$  = angle between ejection path and vertical axis at time of ejection

$\varphi$  = expansion ratio ( $= \frac{X_m}{X_0} = \frac{v_m}{v_0}$ )

#### Subscripts

0 = initial conditions (defined by  $V_0 = 0$ )

p = peak pressure conditions

b = "burnt" conditions (defined by  $N = C$ )

m = "muzzle" conditions

bp = black powder

#### METHOD OF SOLUTION

##### Equations for the System

For convenience of calculation, the airplane is taken as the frame of reference and all forces and accelerations are summed along the path of ejection.

Figure 1 illustrates the forces acting upon the ejected mass when the ejection path makes an angle  $\theta$  with the vertical axis. The load factor,  $\alpha$ , is the ratio of the apparent weight to the actual weight of the ejected mass just prior to ejection. This apparent weight is due to the combined effects of the gravitational and inertial forces. It can be shown that  $\alpha$  is also the acceleration in g's of the ejected mass with respect to the airplane as a result of these inertial and gravitational forces during ejection.

The equation of motion of the ejected mass with respect to the airplane is then

$$F = M\dot{V} - \alpha Mg + F_f \quad (1)$$

A second equation of motion is

$$V = \dot{X} \quad (2)$$

The equation of motion of the ejected mass with respect to the earth is

$$PA - F_f \pm Mg \cos \theta = M\dot{V}_{M/G}^{(7)} \quad (3)$$

Since it is generally true that

$$\dot{V}_{M/G} = \dot{V} + \dot{V}_{P/G} \quad (4)$$

We may find  $\dot{V}_{P/G}$  as the difference between equations (1) and (3). This gives

$$\dot{V}_{P/G} = \dot{V}_{M/G} - \dot{V} = g(\pm \cos \theta - \alpha)^{(7)} \quad (5)$$

Therefore, if the acceleration of the mass with respect to the airplane,  $\dot{V}$ , is found with the aid of the computer, the acceleration of the mass with respect to the earth,  $\dot{V}_{M/G}$ , can be found from:

$$\dot{V}_{M/G} = \dot{V} + g(\pm \cos \theta - \alpha)^{(7)} \quad (6)$$

This quantity is an important aeromedical parameter.

The ballistic equations are:

Equation of state

$$\frac{Pv}{12} = FX = NRT \quad (7)$$

Energy equation

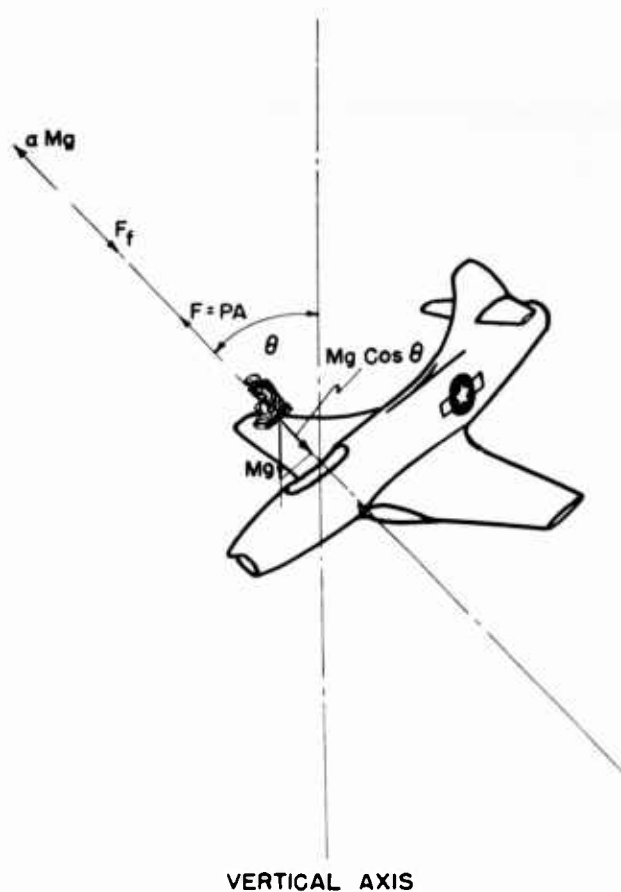
$$NC_v T_a = NC_v T + 1/2 MV^2 + \frac{\beta}{2} MW^2 - \alpha Mg(X - X_0) + \int_{X=X_0}^X F_f dX \quad (8)$$

Equation of burning

$$\frac{d}{dt} \left( \frac{N}{C} \right) = \frac{BP}{W} \quad (9)$$

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<sup>(7)</sup> Plus for downward ejection, minus for upward ejection.



VERTICAL AXIS

Figure 1. Summation of forces on ejected mass

Differentiation of equation (7) and combination with equation (2), after some rearrangement,<sup>(8)</sup> results in:

$$CF^* \frac{d}{dt} \left( \frac{N}{C} \right) = XF^* + \bar{\gamma} FV + (\bar{\gamma} - \gamma) V(aMg - F_f) \quad (10)$$

This may be combined with equation (9) to give:

$$QF = XF^* + \bar{\gamma} FV + (\bar{\gamma} - \gamma) V(aMg - F_f) \quad (11)$$

where

$$Q = \frac{CBF^*}{WA} \quad (12)$$

It may be noted that equation (11) is valid where  $F_f$  is any function of  $X$ .

<sup>(8)</sup> See Appendix II for derivation of equation 10.

The solution of a specific catapult system under specific operating conditions can be achieved if six constants and three initial conditions are known. The equations used for the computer are:

$$F = M\dot{V} - \alpha Mg + F_f \quad (1)$$

$$V = \dot{X} \quad (2)$$

$$Q\dot{F} = X\dot{F} + \bar{\gamma}FV + (\bar{\gamma} - \gamma)V(\alpha Mg - F_f) \quad (11)$$

The constants for the system are  $M$ ,  $\alpha$ ,  $K$ ,  $Q$ ,  $\gamma$ , and  $\bar{\gamma}$ . The initial conditions for the system are  $F_0$ ,  $X_0$ , and  $V_0 = 0$ .

Standard difference methods are used with a high-speed digital computer. A Remington Rand model No. 409-2A computer has been employed for this problem at Frankford Arsenal.

The output of the computer tabulates values of  $X$ ,  $V$ , and  $F$  for each time increment. (An increment of five milliseconds has been found to be satisfactory.) The solution is halted when  $X$  attains the value of  $X_m$ .<sup>(9)</sup> The time required for the stroke and the velocity at muzzle,  $V_m$ , may be noted. The peak pressure,  $P_p$ , is obtained from the greatest value of thrust, since  $P_p = F_p/A$ . The maximum acceleration of the mass,  $\dot{V}$ , may be obtained from the greatest difference in successive values of velocity since  $\dot{V} \approx \Delta V/\Delta T$ . The maximum absolute acceleration of the mass,  $\dot{V}_{M/G}$ , may then be found with equation (6).

#### CONSTANTS AND INITIAL CONDITIONS

The constants and initial conditions for a given case may be evaluated as follows:

1.  $M$  is the ejected mass in slugs.
2.  $\alpha$  is the load factor just prior to ejection or the ratio of the apparent weight to the actual weight,  $Mg$ , of the ejected mass just prior to ejection. Also,  $\alpha$  is the acceleration in g's of the ejected mass relative to the airplane as a result of the inertial and gravitational forces during ejection. By convention, a positive value of  $\alpha$  represents an acceleration of the ejected mass relative to the airplane that acts to assist the ejection, while a negative value of  $\alpha$  represents an acceleration of the mass relative to the airplane that acts to retard the ejection.

<sup>(9)</sup>The methods for calculating  $X_0$  and  $X_m$  are given in Appendix I.

3.  $K$  is the frictional drag coefficient defined by the relation:

$$F_f = K(X - X_0)^2 \quad (13)$$

The value of  $K$  for a given system flying at a given altitude at a given speed is a function of the aerodynamic drag on the ejected mass, the guide rail design geometry, and the coefficient of rail friction. For a given guide rail design and coefficient of friction, the aerodynamic drag as a function of stroke may be determined. This can be done by wind tunnel tests or aerodynamic calculations. The frictional drag,  $F_f$ , may then be determined as a function of stroke. The constant,  $K$ , is then chosen by the usual means to give the best fit to the approximately parabolic curve of  $F_f$  vs  $X$ .<sup>(10)</sup>

4.  $Q$  is the ballistic constant defined by:

$$Q = \frac{CBF^*}{WA} \quad (12)$$

The values of  $C$ ,  $W$ , and  $A$  are known for a given system. The  $BF^*$  product is a function of propellant composition. It is determined from a semi-empirical calibration procedure described in the next section.

5.  $\bar{\gamma}$  is the pseudo-ratio of specific heats defined by the relationship

$$\bar{\gamma} - 1 = (1 + \beta)(\gamma - 1) \quad (14)$$

$\gamma$  is taken as 1.25 for propellant combustion products. The ratio of heat loss to kinetic energy,  $\beta$ , is taken as 1:3. This is a common assumption for guns and will be used until experimental data become available.

6.  $X_0$  is the initial length of the equivalent catapult. It is defined as

$$X_0 = \frac{v_0}{12A} \quad (15)$$

7.  $F_0$  is the initial gas force at the instant of motion. It is found in either of two ways:

(a) For  $\alpha < 0$ , the initial force may be taken as  $-\alpha Mg$ .

<sup>(10)</sup> There may be cases where the friction function is best fitted by an expression other than the commonly used parabolic one. The expression that best fits the data should be used (see Footnote (2)).

(b) For  $a \geq 0$ , the initial force may be taken as the force developed by burning the booster charge of black powder instantaneously in the initial volume. Then: <sup>(11)</sup>

$$F_0 = P_0 A = \frac{12 C_{bp} F^*_{bp} A}{v_0} = \frac{C_{bp} F^*_{bp}}{X_0} \quad (16)$$

where  $F^*_{bp}$  is taken as 80,000 ft-lb/lb.

### CALIBRATION PROCEDURE

The purpose of the calibration procedure is to determine the  $BF^*$  product for a particular composition of propellant when burned in a propellant actuated device. <sup>(12)</sup> A semi-empirical technique is employed. Firing data for a standard catapult are used. The solution for that catapult is generated on the computer for an assumed value of  $Q$ , and the muzzle velocity thus obtained,  $V_m$ , is noted. Several tries of  $Q$  are made until  $V_m$  agrees with the experimental data. The value of  $Q$  for this trial is taken as the proper value of  $Q$ . As a further check, values of maximum thrust and time of stroke should approximately agree with experimental values. <sup>(13)</sup>

The experimental quantities  $C$ ,  $W$ , and  $A$  are known, and  $Q$  has been obtained from a computer match of muzzle velocity. The  $BF^*$  product may therefore be found from the relationship:

$$Q = \frac{CBF^*}{WA}$$

The value of  $BF^*$  is a property of the propellant composition and may be used to help evaluate  $Q$  for a new catapult that will use this "calibrated" propellant.

If the  $BF^*$  product is determined in the above manner from two or more catapults that use the same composition propellant, then a mean value of the  $BF^*$  products should be taken as the most reliable value.

(11) Future experimental work may show that the initial gas force,  $F_0$ , for zero or positive "g" firings is more closely dependent upon the cartridge case rupture pressure than the black powder charge weight.

In any case, it may be advisable to shift the point  $t = 0$  for cases of large positive  $a$  from the time where  $V = 0$  to the time where the cartridge case ruptures. Since there is a delay time,  $t_d$ , from when the latches are released to when the cartridge case ruptures, the original volume will have increased and the tube velocity will be greater than zero due to the accelerating field  $ag$ . For  $t = 0$  at cartridge case rupture, we have  $V_0 = agt_d$  and new  $v_0 = \text{old } v_0 + \Delta v$  where  $\Delta v = \frac{1}{2} agt_d^2$ . Future experimental work is necessary to ascertain  $t_d$ .

(12) This will probably differ from the  $BF^*$  product of the propellant when burned in a closed bomb or vented vessel.

(13) Computer calibrations and computer curves for several standardized catapults, together with all important constants, are presented in Appendix III.

## EXTENSION OF SOLUTION

After the performance of a proposed catapult has been evaluated with the computer and found to be satisfactory for the normal operating conditions (i. e.,  $\alpha = -1$ ,  $K = 0$ ), it is very helpful to study the catapult performance under combinations of g forces and frictional forces that may be expected in service. Changes in the ejection velocity,  $V_a$ , the maximum acceleration,  $\dot{V}$  max; the peak pressure,  $P_p$ ; and the ejection time,  $t_a$ , are observed. This knowledge is important for rapid, inexpensive development of a new catapult.

The solutions obtained from the computer analysis all assume that the web size is sufficient to insure burning until tube separation. This is so because the term  $\frac{d}{dt}\left(\frac{N}{C}\right)$  is eliminated between two ballistic equations, (5) and (6), and is never entered into the computer equations. Hence, the computer has no way of checking when the charge has been consumed ( $N = C$ ).

If the web thickness is sufficiently large, this does not pose a problem. On the other hand, if the web is of just sufficient size to support burning until tube separation for a normal ejection, the grains may be consumed at some time prior to tube separation in cases of large g forces and large frictional forces retarding the motion. This is because the burning rate is proportional to the pressure, and the pressure (or force,  $F$ ) is higher when large external forces retard the motion. When this condition occurs, an estimation of muzzle velocity,  $V_a$ , with "burnout" may be made.

The burning law may be written in the form

$$\frac{d}{dt}\left(\frac{N}{C}\right) = \frac{Q}{CF^*} F \quad (17)$$

This can be integrated from  $t = 0$  to  $t = t_b$  (where  $N/C = 1$ ) and rearranged to give

$$\int_{t=0}^{t=t_b} F dt = (1 - N_0/C) \frac{CF^*}{Q} \quad (18)$$

Hence, the condition of "burnt" can be expressed by equating an area under the  $F$ - $t$  curve to a constant.



The term  $N_0/C$  in equation (18) is a measure of the charge burned at the instant of motion. This will increase for larger negative (retarding) values of  $\alpha$  and can be found from

$$\frac{N_0}{C} = \frac{-\alpha M_g X_0}{C F^*} - \frac{C_{bp} F^*_{bp}}{C F^*} \quad (19)$$

The second term on the right side of equation (19) accounts for the black powder's contribution to  $F_0$ .

In order to use equation (18), one must make some reasonable assumption from available data about either  $B$ ,  $F^*$ , or the point of "burnt." For the solutions shown in Appendix III, it was assumed that either  $V_m = V_b$  when  $\alpha = -1$  or a reasonable value of  $F^*$  was selected. The resulting calculated points of "burnt" are shown on the curves in Appendix III where broken lines start at "burnt" and continue to the "muzzle" point. These broken lines are the solutions that would have been obtained if the burn-out did not occur until the "muzzle" point. A stepwise numerical solution of the M1A1 catapult from "burnt" to "muzzle" is shown for the case of  $\alpha = -7$  to illustrate the expected real performance when  $V_m > V_b$ .

The solution from "burnt" to "muzzle" is described by a pseudo-adiabatic expansion.

$$P_b v_b^{\bar{\gamma}} = P_m v_m^{\bar{\gamma}} \quad (20a)$$

or

$$F_b X_b^{\bar{\gamma}} = F_m X_m^{\bar{\gamma}} \quad (20b)$$

$F_m$  is found from this relationship since the other quantities are known. The work done by the gas during this expansion is:

$$E = \frac{F_m X_m - F_b X_b}{1 - \bar{\gamma}} \quad (21)$$

This energy is used to overcome the rail friction and to accelerate the catapult according to the following relationship:

$$E = \int_{X-X_b}^{X_m} F_f dX - \alpha M_g (X_m - X_b) + 1/2 M (V_m^2 - V_b^2) \quad (22)$$

For a known friction function, this equation can be solved for  $V_m$ .

**APPENDIX I**

# CONVERSION FROM ACTUAL TO EQUIVALENT CATAPULT

The actual catapult of several concentric tubes is reduced to an equivalent catapult of constant bore for convenient analysis. The reference bore is determined by that tube in motion at the moment of maximum thrust.

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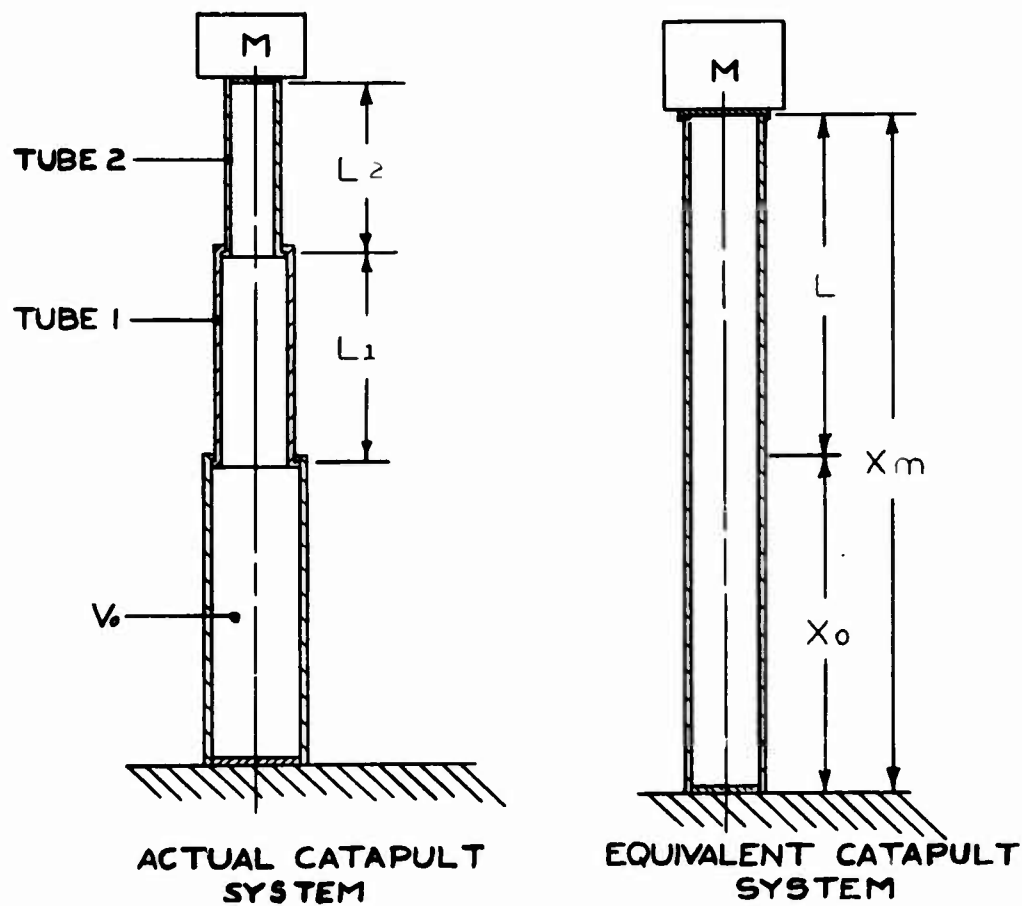


Figure 2. Diagrammatic representation of actual and equivalent catapult systems

Figure 2 schematically represents an actual catapult system and its ballistic equivalent for the case of maximum thrust occurring during motion of tube 1. The following symbols are used:

$V_0$	= initial volume of actual catapult (including the annulus volumes)	in. <sup>3</sup>
$A_1$	= effective bore area of tube 1. This is determined by the outside diameter of tube 1 at its center.	in. <sup>2</sup>
$A_2$	= effective bore area of tube 2, determined in a manner similar to $A_1$ .	in. <sup>2</sup>
$A$	= bore area of equivalent catapult (shown here as $A_1$ ).	in. <sup>2</sup>
$\ell_1$	= stroke length of tube 1	ft
$\ell_2$	= stroke length of tube 2	ft
$L$	= stroke length of equivalent catapult	ft
$X_0$	= initial length of equivalent catapult	ft
$X_m$	= total length of equivalent catapult ( $= X_0 + L$ )	ft

For  $A = A_1$ :

$$L = \ell_1 + \frac{A_2}{A_1} \ell_2$$

$$X_0 = \frac{v_0}{12A_1}$$

$$X_m = X_0 + L$$

For  $A = A_2$ :

$$L = \frac{A_2}{A_1} \ell_1 + \ell_2$$

$$X_0 = \frac{v_0}{12A_2}$$

$$X_m = X_0 + L$$

These equations may be extended for three or more tubes.

APPENDIX II

# DERIVATION OF EQUATION (10)

$$\frac{P_v}{12} = FX = NRT \quad (7)$$

$$NC_v T_a = NC_v T + 1/2 MV^2 + \frac{\beta}{2} MV^2 - \alpha Mg(X - X_0) + \int_{X=X_0}^X F_f dX \quad (8)$$

Differentiating equation (7) with respect to time, we have

$$XF + FV = CRT_a \left[ \frac{N}{C} \frac{d}{dt} \left( \frac{T}{T_a} \right) + \frac{T}{T_a} \frac{d}{dt} \left( \frac{N}{C} \right) \right] \quad (A)$$

Rearrangement of equation (8) results in

$$\frac{T}{T_a} = 1 - \frac{(\bar{\gamma} - 1) MV^2}{2NRT_a} - \frac{(\gamma - 1) \int F_f dX}{NRT_a} + \frac{(\gamma - 1) \alpha Mg (X - X_0)}{NRT_a} \quad (B)$$

Since

$$C_v = \frac{R}{\gamma - 1} \quad (C)$$

And

$$\bar{\gamma} - 1 = (1 + \beta)(\gamma - 1) \quad (D)$$

Substitution of  $T/T_a$  from equation (B) into equation (A) gives

$$\begin{aligned} XF + FV = CF^* \left\{ \frac{N}{C} \frac{d}{dt} \left[ 1 - \frac{(\bar{\gamma} - 1) MV^2}{2NF^*} - \frac{(\gamma - 1) \int F_f dX}{NF^*} \right. \right. \\ \left. \left. + \frac{(\gamma - 1) \alpha Mg (X - X_0)}{NF^*} \right] + \left[ 1 - \frac{(\bar{\gamma} - 1) MV^2}{2NF^*} - \frac{(\gamma - 1) \int F_f dX}{NF^*} \right. \right. \\ \left. \left. + \frac{(\gamma - 1) \alpha Mg (X - X_0)}{NF^*} \right] \frac{d}{dt} \left( \frac{N}{C} \right) \right\} \quad (E) \end{aligned}$$

Where

$$RT_a = F^* \quad (F)$$

When the indicated differentiation is performed, equation (E) takes the form

$$\begin{aligned}
 \dot{X}\bar{F} + F\dot{V} = NF^* & \left\{ - \left( \frac{(\bar{\gamma} - 1) M}{2F^*} \right) \left( \frac{2NV\dot{V} - V^2\dot{N}}{N^2} \right) - \frac{(\gamma - 1)}{F^*} \left( \frac{N \frac{d}{dt} \int F_f dX - \dot{N} \int F_f dX}{N^2} \right) \right. \\
 & \left. + \frac{(\gamma - 1) \alpha M_g}{F^*} \left( \frac{NV - (X - X_0) \dot{N}}{N^2} \right) \right\} \\
 & + NF^* \left\{ 1 - \frac{(\bar{\gamma} - 1) M^2}{2NF^*} - \frac{(\gamma - 1) \int F_f dX}{NF^*} + \frac{(\gamma - 1) \alpha M_g (X - X_0)}{NF^*} \right\}
 \end{aligned} \quad (G)$$

Where

$$\frac{d}{dt} \int F_f dX = \frac{d}{dX} \int F_f dX \frac{dX}{dt} = F_f V \quad (H)$$

Expansion and reduction of (G) results in

$$\dot{X}\bar{F} + F\dot{V} = - (\bar{\gamma} - 1) M V \dot{V} - (\gamma - 1) F_f V + (\gamma - 1) \alpha M_g V + NF^* \quad (I)$$

Substituting for  $\dot{V}$  in equation (I) according to equation (1), i. e.;

$$\dot{V} = \frac{1}{M} (F + \alpha M_g - F_f), \text{ simplifies equation (I) to}$$

$$NF^* = \dot{X}\bar{F} + \bar{\gamma} F V + (\bar{\gamma} - 1) V (\alpha M_g - F_f) - (\gamma - 1) V (\alpha M_g - F_f) \quad (J)$$

Which can be further simplified to

$$CF^* \frac{d}{dt} \left( \frac{N}{C} \right) = \dot{X}\bar{F} + \bar{\gamma} F V + (\bar{\gamma} - \gamma) V (\alpha M_g - F_f) \quad (10)$$

This is the required relationship.

APPENDIX III



Computer curves for several catapults with g loads that retard the motion are shown in the attached graphs (Figures 3 to 9, incl). The constants used for the calculations are summarized in the following chart.

<u>Quantity</u>	<u>Unit</u>	<u>M1A1 and M5</u>	<u>M2A1</u>	<u>M3</u>	<u>M4 (M37 cartridge)</u>	<u>T10</u>	<u>T14</u>
Mg	lb	312.0	311.8	365.2	324.0	363.4	300.0
$\alpha$		-1, -3, -5, -7	-1, -3, -5, -7	-1, -3, -5, -7	-1, -3, -5, -7	-1, -3, -5, -7	-1, -3, -5, -7
K	lb/ft <sup>2</sup>	0	0	0	0	0	0
Q	fps	54.0	43.2	110.4	44.0	80.0	26.8
$\bar{\gamma}$		1.33	1.33	1.33	1.33	1.33	1.33
F <sub>0</sub>	lb	$\alpha Mg$	$\alpha Mg$	$\alpha Mg$	$\alpha Mg$	$\alpha Mg$	$\alpha Mg$
X <sub>0</sub>	ft	2.273	2.160	7.763	3.296	3.929	1.130
X <sub>m</sub>	ft	7.325	6.493	15.867	7.531	9.417	2.883
X <sub>m</sub> /X <sub>0</sub>		3.22	3.005	2.044	2.285	2.400	2.55
Propellant composition		M6	M6	H8	H8	T8	M2
Calc BF*	in/sec/psi × ft-lb/lb	105.3	100.8	66.8	75.7	110.0	130.5
C	lb	0.1863	0.1700	0.441	0.1214	0.353	0.0618
W <sub>1</sub>	in.	0.1270	0.1270	0.110	0.118	0.130	0.170
A	in. <sup>2</sup>	2.86	3.125	2.426	1.77	3.733	1.77

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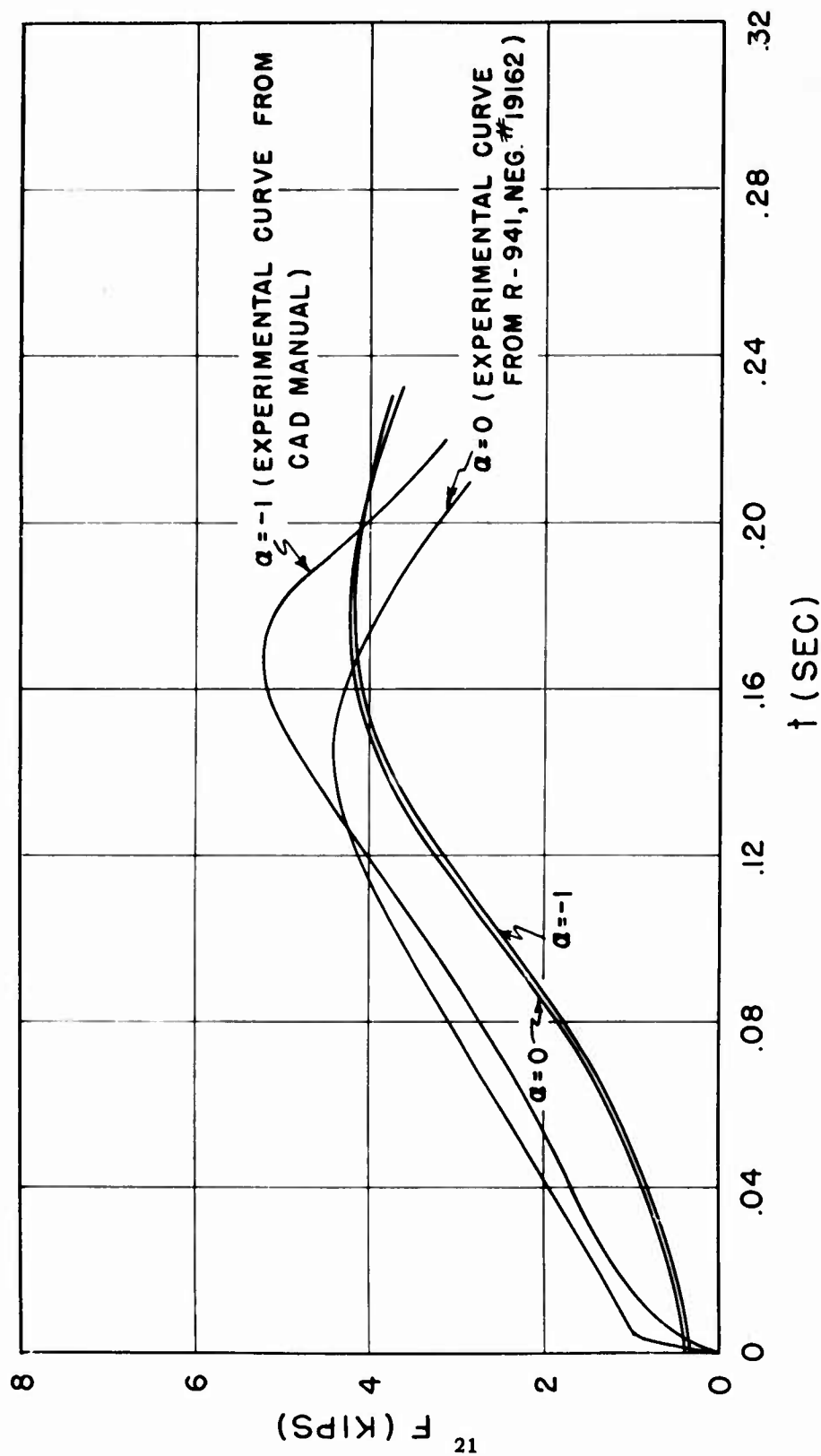


Figure 3. A comparison of computer curves with experimental curves for Catapult, Aircraft Personnel, MIAI

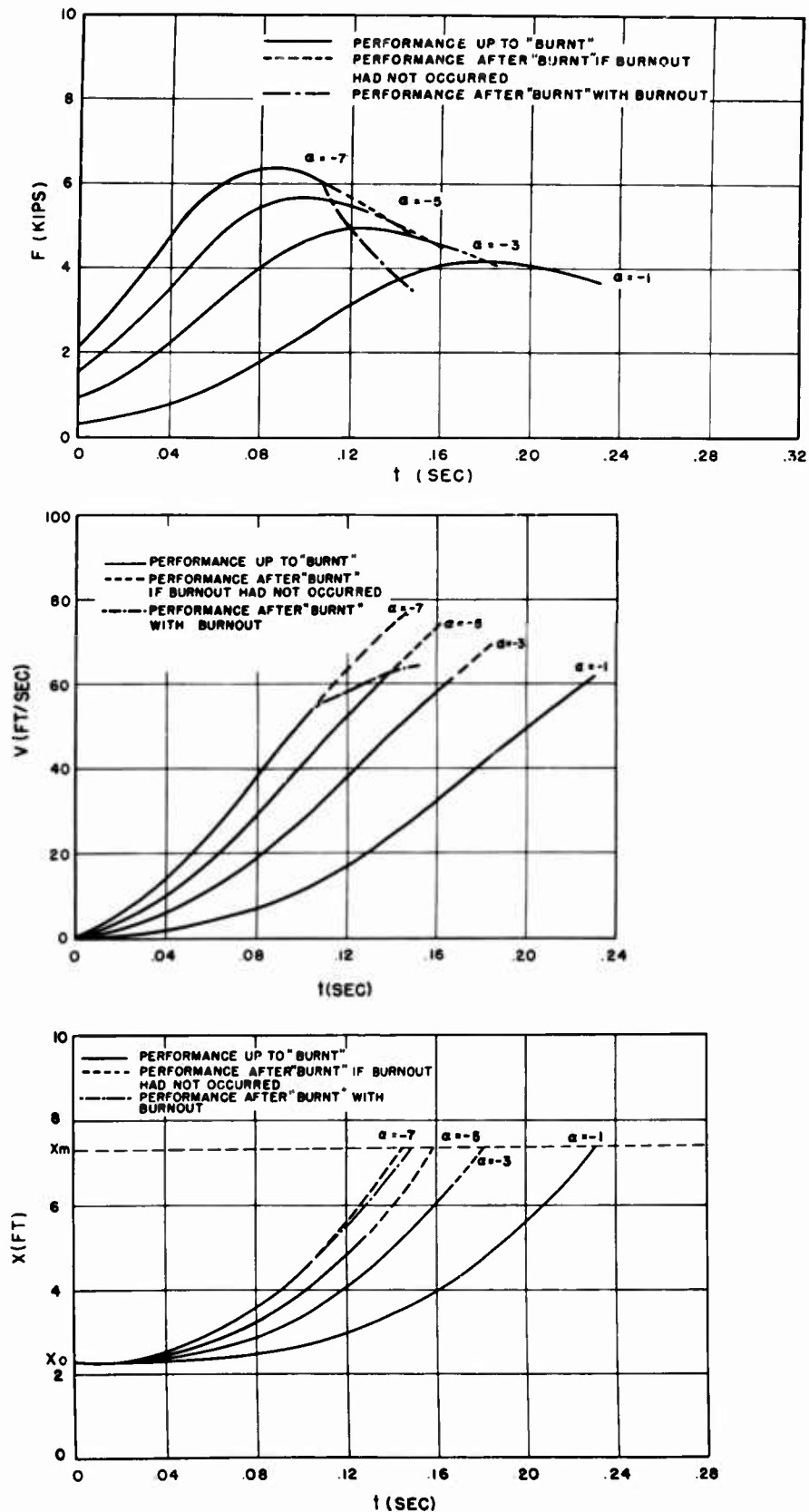


Figure 4. Computer curves of thrust, velocity, and displacement vs time for Catapult Aircraft Personnel, M3

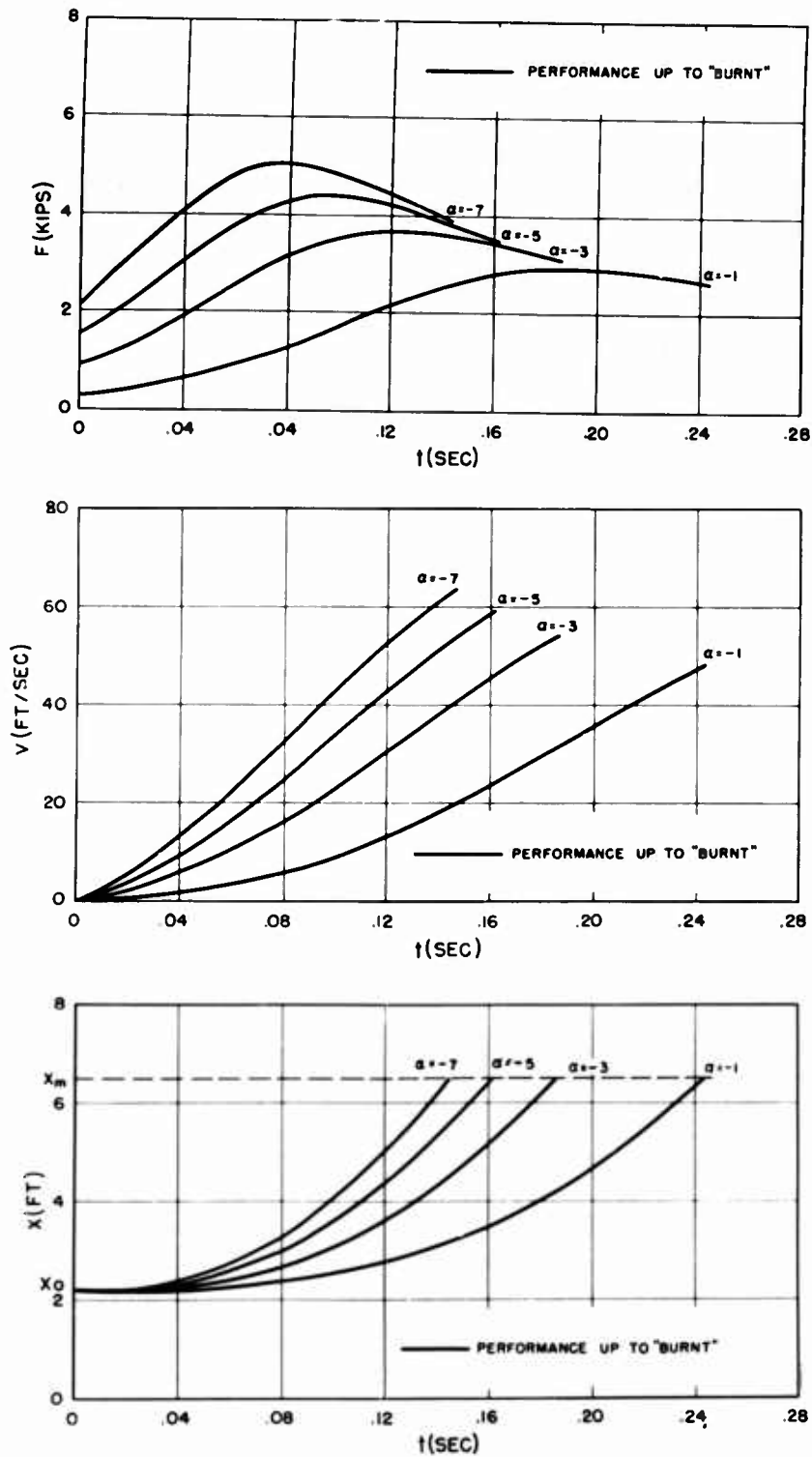


Figure 5. Computer curves of thrust, velocity, and displacement vs time for Catapult, Aircraft Personnel, M2A1

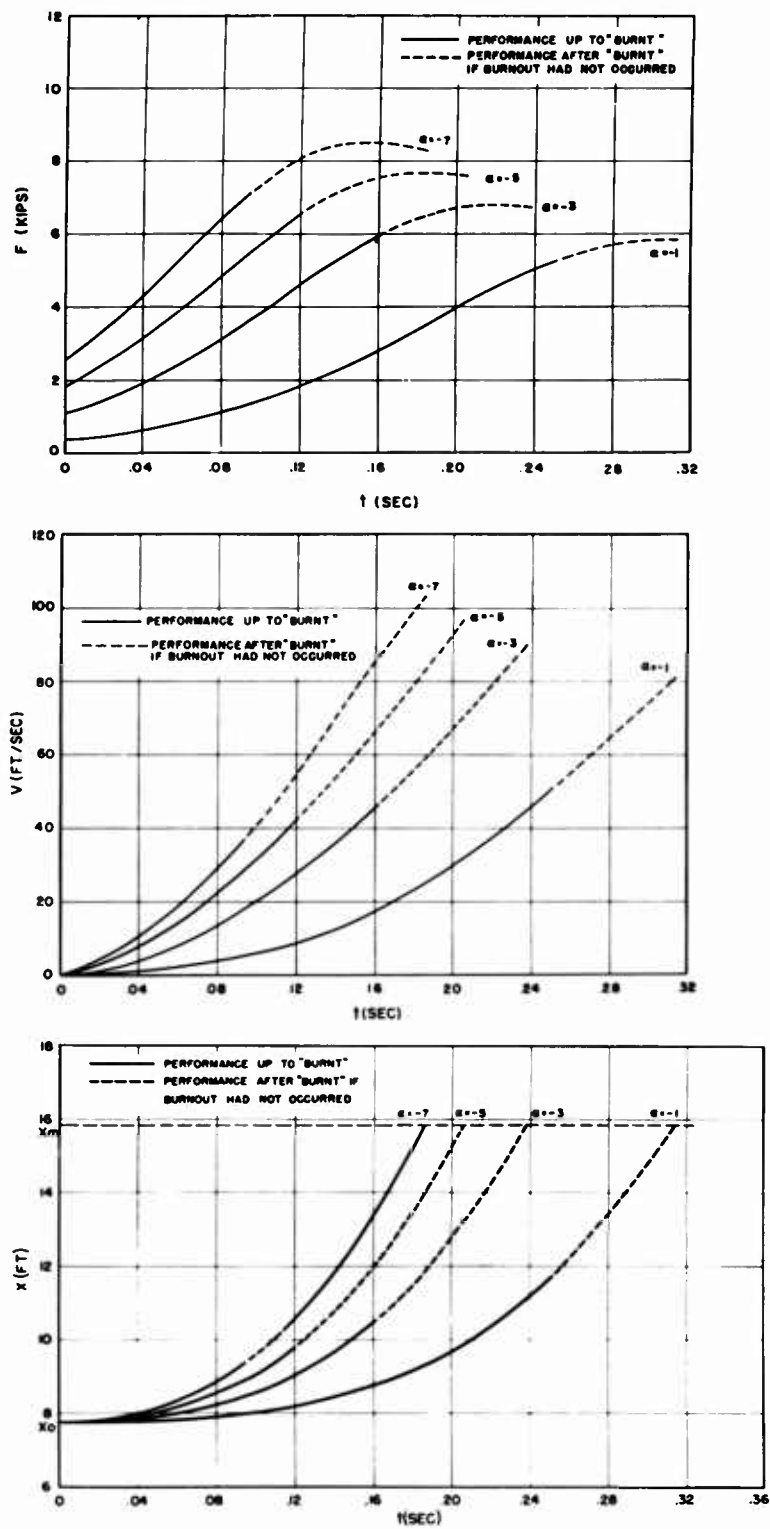


Figure 6. Computer curves of thrust, velocity, and displacement vs time for Catapult, Aircraft Personnel, M3

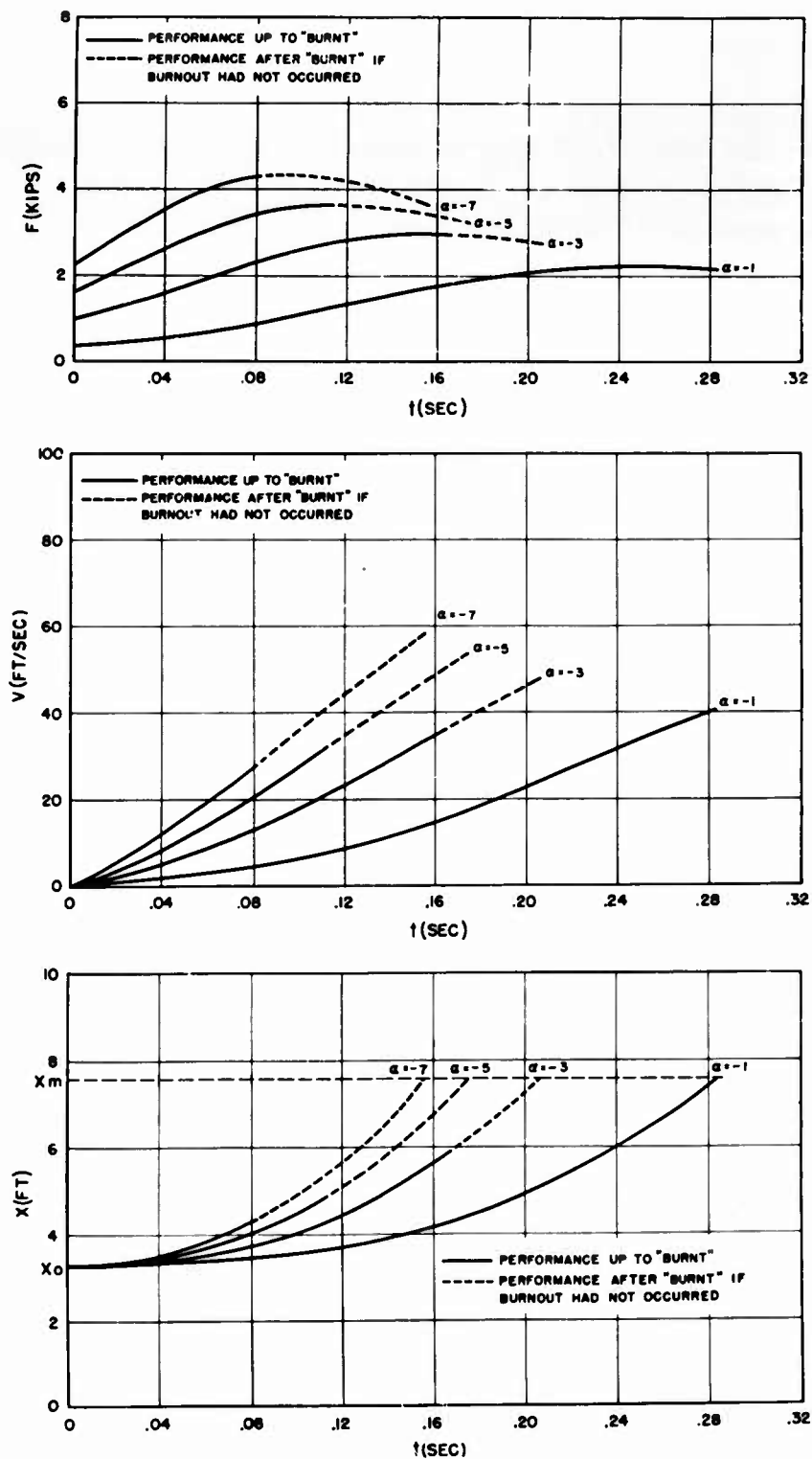


Figure 7. Computer curves of thrust, velocity, and displacement vs time for Catapult, Aircraft Personnel, M4

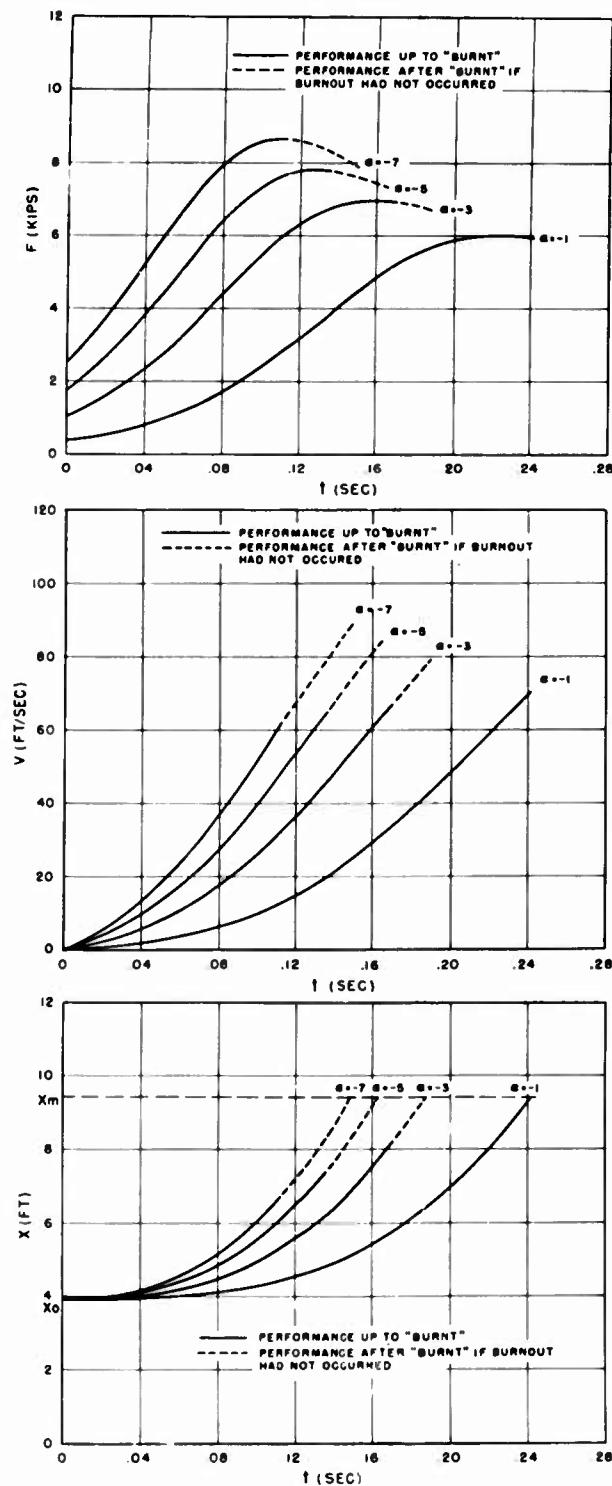


Figure 8. Computer curves of thrust, velocity, and displacement vs time for Catapult, Aircraft Personnel, T10

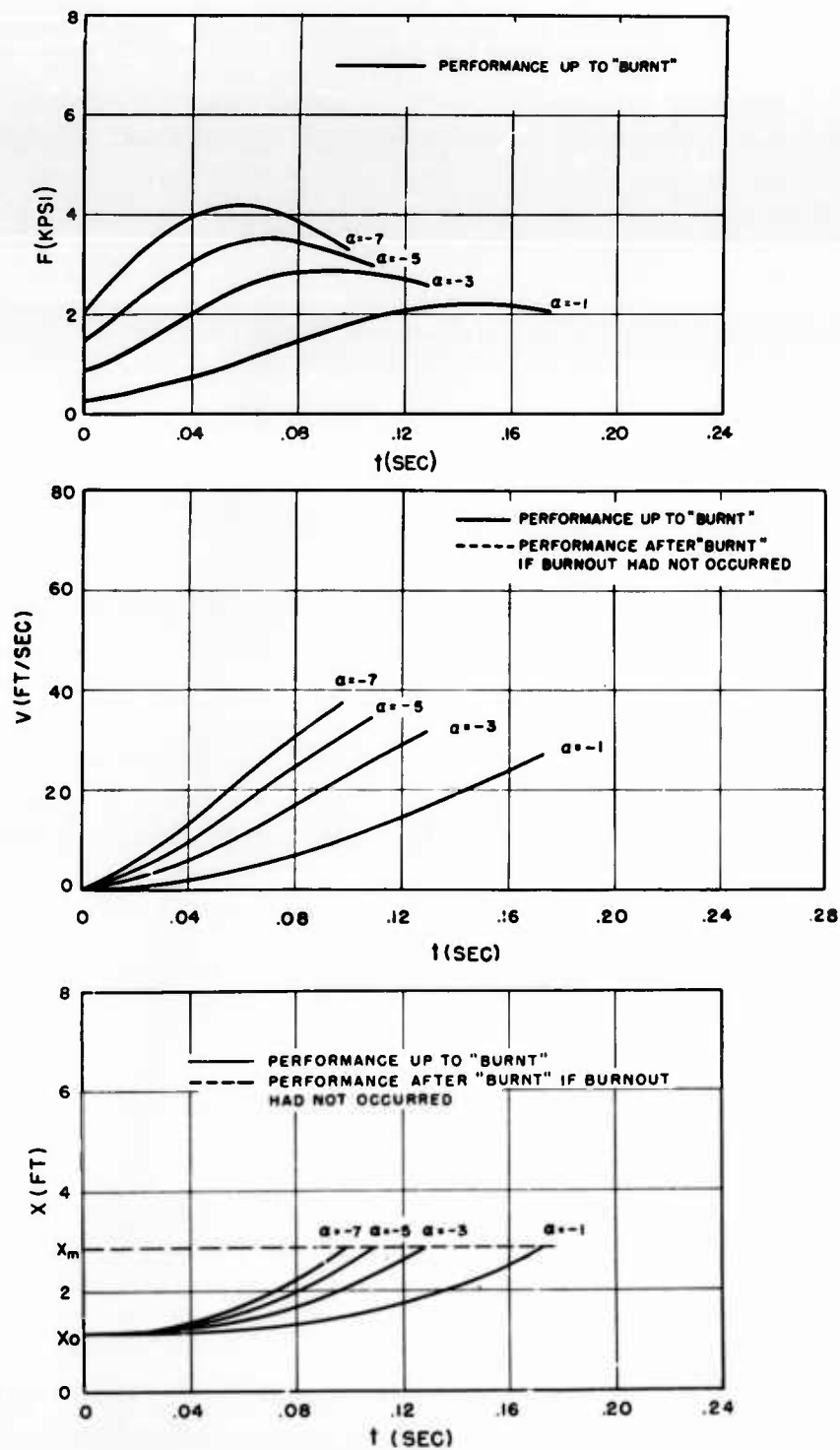


Figure 9. Computer curves of thrust, velocity, and displacement vs time for Catapult, Aircraft Personnel, T14



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